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Gravity-assisted emergent higgs mechanism in the post-inflationary epoch*

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We consider a nonstandard model of gravity coupled to a neutral scalar "inflaton" as well as to $SU(2) \times U(1)$ iso-doublet scalar with positive mass squared and without self-interaction, and to $SU(2) \times U(1)$ gauge fields. The principal new ingredient is employing two alternative non-Riemannian spacetime volume-forms (covariant integration measure densities) independent of the metric. The latter have a remarkable impact — although not introducing any additional propagating degrees of freedom, their dynamics triggers a series of important features: appearance of infinitely large flat regions of the effective "inflaton" potential as well as dynamical generation of Higgs-like spontaneous symmetry breaking effective potential for the $SU(2) \times U(1)$ iso-doublet scalar.

Keywords: Non-Riemannian volume forms; quintessential evolution; dynamical generation of electroweak symmetry breaking.

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1. Introduction

In a remarkable paper from 1986[1] Bekenstein proposed the intriguing idea about a gravity-assisted spontaneous symmetry breaking of electro-weak (Higgs) type without invoking unnatural (according to Bekenstein's opinion) ingredients like negative mass squared and a quartic self-interaction for the Higgs field. By considering a

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model of gravity interacting with a standard Klein–Gordon field (with small positive mass squared and without self-interaction) coupled conformally to the scalar curvature he managed to obtain a prototype of dynamically induced Higgs-like spontaneous symmetry breaking scalar potential. A similar approach was further worked out in Ref. 2.

Motivated by Bekenstein's idea, in this paper we will consider a nonstandard model of gravity coupled to a neutral scalar "inflaton" φ as well as to a $U(1)$-charged $SU(2)$ iso-doublet scalar field σ with a standard positive mass squared and no self-interaction, as well as to $SU(2) \times U(1)$ gauge fields. The essential nonstandard feature of this model is employing non-Riemannian spacetime volume forms — alternative generally covariant integration measure densities defined in terms of auxiliary antisymmetric tensor gauge fields independent of the pertinent Riemannian metric.[a] Although being almost pure-gauge degrees of freedom (see the remark in Sec. 2), the non-Riemannian spacetime volume forms trigger a series of important features unavailable in ordinary gravity-matter models with the standard Riemannian volume-form (given by the square-root of the determinant of the Riemannian metric): (i) The "inflaton" φ develops a remarkable effective scalar potential in the Einstein frame possessing an infinitely large flat region for large negative φ describing the "early" universe evolution; (ii) In the absence of the $SU(2) \times U(1)$ iso-doublet scalar field, the "inflaton" effective potential has another infinitely large flat region for large positive φ describing the "late" post-inflationary (dark energy dominated) universe; (iii) Inclusion of the $SU(2) \times U(1)$ iso-doublet scalar field σ introduces a drastic change in the total effective scalar potential in the post-inflationary universe — the effective potential as a function of σ acquires exactly the electro-weak Higgs-type spontaneous symmetry breaking form.

2. Gravity-Gauge-Field-Matter Model With Two Independent Non-Riemannian Volume-Forms

Our starting point is the following nonstandard gravity-gauge-field-matter system with an action of the general form involving two independent non-Riemannian volume-forms generalizing the model studied in Ref. 6 and whose gauge-field-matter part has an internal $SU(2) \times U(1)$ gauge symmetry (for simplicity we will use units where the Newton constant is taken as $G_{\text{Newton}} = 1/16\pi$):

$$S = \int d^4x \Phi_1(A) \left[R - 2\Lambda_0 \frac{\Phi_1(A)}{\sqrt{-g}} + L^{(1)} \right] + \int d^4x \Phi_2(B) \left[L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} \right]. \quad (1)$$

In (1) and in the sequel the following notations are used:

[a]The method of non-Riemannian spacetime volume forms, originally proposed in Refs. 3 and 4, has been employed in a broad variety of gravity-matter, supergravity and strings/branes models to provide plausible solutions for dynamical generation of cosmological constant, the supersymmetric Higgs effect,[5] unified description of early universe inflation and present day dark energy,[6] unified description of dark energy and dark matter as different manifestations of a single entity[7,8]

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, i.e. alternative generally covariant integration measure densities on the underlying spacetime manifold:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda}, \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda}, \tag{2}$$

  defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields. $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$ in terms of the spacetime metric $g_{\mu\nu}$.
- $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma^\mu_{\nu\lambda}$ is *a priori* independent of the metric $g_{\mu\nu}$. Λ_0 is a small parameter later to be identified with the present epoch small observable cosmological constant.
- $L^{(1)}$ is the sum of two scalar field Lagrangians:

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V_1(\varphi) - g^{\mu\nu}\left(\nabla_\mu\sigma_a\right)^*\nabla_\nu\sigma_a - V_0(\sigma), \tag{3}$$

  where φ denotes the neutral "inflaton" inert under $SU(2)\times U(1)$ and $\sigma \equiv (\sigma_a)$ is a complex $SU(2)\times U(1)$ iso-doublet scalar field with the isospinor index $a = +, 0$ indicating the corresponding $U(1)$ charge. The gauge-covariant derivative in (3) acting on the iso-doublet scalar σ reads:

$$\nabla_\mu\sigma = \left(\partial_\mu - \frac{i}{2}\tau_A\mathcal{A}^A_\mu - \frac{i}{2}\mathcal{B}_\mu\right)\sigma, \tag{4}$$

  with $\frac{1}{2}\tau_A$ (τ_A — Pauli matrices, $A = 1,2,3$) indicating the $SU(2)$ generators and $\mathcal{A}^A_\mu$ ($A = 1,2,3$) and $\mathcal{B}_\mu$ denoting the corresponding $SU(2)$ and $U(1)$ gauge fields. The pertinent scalar field potentials are:

$$V_1(\varphi) = f_1\exp\{-\alpha\varphi\}, \quad V_0(\sigma) = m_0^2\sigma_a^*\sigma_a, \tag{5}$$

  where α, f_1 are dimensionful positive parameters, and $V_0(\sigma)$ is just the standard mass term for the iso-doublet σ_a with positive mass squared.
- $L^{(2)}$ is the sum of a second "inflaton" Lagrangian plus the canonical Lagrangians for the $SU(2)$ and $U(1)$ gauge fields $\mathcal{A}^A_\mu$, $\mathcal{B}_\mu$:

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + U(\varphi) - \frac{1}{4g^2}F^2(\mathcal{A}) - \frac{1}{4g'^2}F^2(\mathcal{B}), \tag{6}$$

  where:

$$U(\varphi) = f_2\exp\{-2\alpha\varphi\}, \tag{7}$$

  with f_2 another dimensionful positive parameter, whereas b is a dimensionless one, and (all indices $A, B, C = (1,2,3)$):

$$F^2(\mathcal{A}) \equiv F^A_{\mu\nu}(\mathcal{A})F^A_{\kappa\lambda}(\mathcal{A})g^{\mu\kappa}g^{\nu\lambda}, \quad F^2(\mathcal{B}) \equiv F_{\mu\nu}(\mathcal{B})F_{\kappa\lambda}(\mathcal{B})g^{\mu\kappa}g^{\nu\lambda}, \tag{8}$$

$$F^A_{\mu\nu}(\mathcal{A}) = \partial_\mu\mathcal{A}^A_\nu - \partial_\nu\mathcal{A}^A_\mu + \epsilon^{ABC}\mathcal{A}^B_\mu\mathcal{A}^C_\nu, \quad F_{\mu\nu}(\mathcal{B}) = \partial_\mu\mathcal{B}_\nu - \partial_\nu\mathcal{B}_\mu. \tag{9}$$

- $\Phi(H)$ indicates the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}, \tag{10}$$

whose presence is crucial for nontriviality of the model.

Remark. The systematic canonical Hamiltonian treatment of (1) (see Appendix A) and, more generally, of any gravity-matter models built with non-Riemannian volume-forms[9] shows that the auxiliary 3-index tensor gauge fields $A_{\mu\nu\kappa}$, $B_{\mu\nu\kappa}$, $H_{\mu\nu\kappa}$ are almost pure-gauge, i.e. they do not introduce any new propagating field degrees of freedom except for a few free integration constants in their respective equations of motion (see (14)–(15)).

Let us note that the requirement for invariance (with the exception of the mass term $V_0(\sigma)$ of the iso-doublet σ_a) under the following global Weyl-scale symmetry:

$$\begin{gathered} g_{\mu\nu} \to \lambda g_{\mu\nu}, \quad \varphi \to \varphi + \frac{1}{\alpha}\ln\lambda, \quad A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa}, \quad B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa}, \\ \Gamma^\mu_{\nu\lambda}, H_{\mu\nu\kappa}, \sigma_a, \mathcal{A}^A_\mu, \mathcal{B}_\mu \text{ — inert}, \end{gathered} \tag{11}$$

uniquely fixes the structure of the non-Riemannian-measure gravity-gauge-field-matter action (1). In particular, for the same reason we have multiplied by an appropriate exponential factor the "inflaton" kinetic term in $L^{(2)}$ (6).

Following Ref. 6 we will now derive the effective Einstein-frame form of the dynamics described by (1).

Variation of the action (1) with respect to affine connection $\Gamma^\mu_{\nu\lambda}$:

$$\int d^4x\sqrt{-g}g^{\mu\nu}\left(\frac{\Phi_1}{\sqrt{-g}}\right)(\nabla_\kappa\delta\Gamma^\kappa_{\mu\nu} - \nabla_\mu\delta\Gamma^\kappa_{\kappa\nu}) = 0 \tag{12}$$

shows that $\Gamma^\mu_{\nu\lambda}$ becomes a Levi-Civita connection $\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}(\partial_\nu\bar{g}_{\lambda\kappa} + \partial_\lambda\bar{g}_{\nu\kappa} - \partial_\kappa\bar{g}_{\nu\lambda})$ with respect to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu}, \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}}. \tag{13}$$

Variation of (1) with respect to auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations (using the short-hand notation χ_1 from (13)):

$$\partial_\mu[R + L^{(1)} - 4\Lambda_0\chi_1] = 0, \quad \partial_\mu\left[L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\right] = 0, \quad \partial_\mu\left(\frac{\Phi_2(B)}{\sqrt{-g}}\right) = 0, \tag{14}$$

whose solutions read:

$$\begin{gathered} \frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const.}, \quad R + L^{(1)} - 4\Lambda_0\chi_1 = -M_1 = \text{const.}, \\ L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const.}. \end{gathered} \tag{15}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 is arbitrary dimensionless integration constants.

Now, varying (1) with respect to $g_{\mu\nu}$ and using relations (15) we obtain:

$$\chi_1 \left[R_{\mu\nu} + \frac{1}{2}(g_{\mu\nu}L^{(1)} - T^{(1)}_{\mu\nu}) - \Lambda_0 \chi_1 g_{\mu\nu} \right] - \frac{1}{2}\chi_2 [T^{(2)}_{\mu\nu} + g_{\mu\nu} M_2] = 0, \tag{16}$$

where χ_1 and χ_2 are the same as in (13) and (15), respectively, and $T^{(1,2)}_{\mu\nu}$ are the energy–momentum tensors of the matter-gauge-field Lagrangians (3)–(6) with the standard definitions: $T^{(1,2)}_{\mu\nu} = g_{\mu\nu}L^{(1,2)} - 2\partial L^{(1,2)}/\partial g^{\mu\nu}$.

Taking the trace of Eq. (16) and using again the second relation (15) we solve for the scale factor χ_1 (13):

$$\chi_1 = 2\chi_2 \frac{\frac{T^{(2)}}{4} + M_2}{\frac{L^{(1)} - T^{(1)}}{2} - M_1}, \quad T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}. \tag{17}$$

Using the second relation (15), the gravity equations (16) can be put in the Einstein-like form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}g_{\mu\nu}(L^{(1)} + M_1 - 2\Lambda_0\chi_1) + \frac{1}{2}(T^{(1)}_{\mu\nu} - g_{\mu\nu}L^{(1)}) + \frac{\chi_2}{2\chi_1}[T^{(2)}_{\mu\nu} + g_{\mu\nu}M_2]. \tag{18}$$

Now, using expression (17) for χ_1 and following the same steps as in Ref. 6 we can bring Eqs. (18) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (13), i.e. the Einstein-frame gravity equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu}, \quad T^{\rm eff}_{\mu\nu} = g_{\mu\nu}L_{\rm eff} - 2\frac{\partial}{\partial g^{\mu\nu}}L_{\rm eff}, \tag{19}$$

with effective energy–momentum tensor corresponding to the following effective Einstein-frame matter Lagrangian:

$$L_{\rm eff} = A(\varphi,\sigma)X + B(\varphi)X^2 - U_{\rm eff}(\varphi,\sigma) - \bar{g}^{\mu\nu}(\nabla_\mu\sigma_a)^*\nabla_\nu\sigma_a - \frac{\chi_2}{4g^2}\bar{F}^2(\mathcal{A}) - \frac{\chi_2}{4g'^2}\bar{F}^2(\mathcal{B}). \tag{20}$$

In (20) the following notations are used:

- X is the standard short-hand notations for the "inflaton" kinetic term:

$$X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi. \tag{21}$$

- The coefficient functions in the "k-essence"-type[10–13] expression (first two terms on the right-hand side of (20) with *nonlinear* functional dependence on X (21)) are given by:

$$A(\varphi,\sigma) \equiv 1 + \frac{1}{2}be^{-\alpha\varphi}\frac{V_1(\varphi) + V_0(\sigma) - M_1}{U(\varphi) + M_2} = 1 + \frac{1}{2}be^{-\alpha\varphi}\frac{f_1 e^{-\alpha\varphi} + m_0^2\sigma_a^*\sigma_a - M_1}{f_2 e^{-2\alpha\varphi} + M_2}; \tag{22}$$

and

$$B(\varphi) \equiv -\frac{1}{4}\frac{\chi_2 b^2 e^{-2\alpha\varphi}}{U(\varphi)+M_2} = -\frac{1}{4}\frac{\chi_2 b^2 e^{-2\alpha\varphi}}{f_2 e^{-2\alpha\varphi}+M_2}. \tag{23}$$

- The full effective scalar field potential reads:

$$U_{\text{eff}}(\varphi,\sigma) \equiv \frac{[V_1(\varphi)+V_0(\sigma)-M_1]^2}{4\chi_2[U(\varphi)+M_2]} + 2\Lambda_0$$
$$= \frac{(f_1 e^{-\alpha\varphi} + m_0^2\sigma_a^*\sigma_a - M_1)^2}{4\chi_2[f_2 e^{-2\alpha\varphi}+M_2]} + 2\Lambda_0, \tag{24}$$

where in (22)–(24) the explicit forms of $V_1(\varphi), V_0(\sigma)$ and $U(\varphi)$ (3)–(6) are inserted.

- In the last line of (20) the Einstein-frame versions of the gauge-field Lagrangians are used, where:

$$\bar{F}^2(\mathcal{A}) \equiv F^A_{\mu\nu}(\mathcal{A})F^A_{\kappa\lambda}(\mathcal{A})\bar{g}^{\mu\kappa}\bar{g}^{\nu\lambda}, \quad \bar{F}^2(\mathcal{B}) \equiv F_{\mu\nu}(\mathcal{B})F_{\kappa\lambda}(\mathcal{B})\bar{g}^{\mu\kappa}\bar{g}^{\nu\lambda}. \tag{25}$$

3. Effective Scalar Potential — Infinitely Large Flat Regions and Emergent Higgs-Like Behavior

A remarkable feature of the effective Einstein-frame scalar potential (24) is that it possesses an infinitely large flat region for large negative φ and is independent of σ there:

$$U_{\text{eff}}(\varphi,\sigma) \simeq U_{(-)} \equiv \frac{f_1^2}{4\chi_2 f_2} + 2\Lambda_0,$$
$$A(\varphi,\sigma) \simeq A_{(-)} \equiv 1 + \frac{1}{2}b\frac{f_1}{f_2}, \quad B(\varphi) \simeq B_{(-)} \equiv -\chi_2\frac{b^2}{4f_2}. \tag{26}$$

Thus, for large negative values of the "inflaton" the effective matter Lagrangian (20) reads:

$$L^{(-)}_{\text{eff}} = A_{(-)}X + B_{(-)}X^2 - U_{(-)} + L[\sigma,\mathcal{A},\mathcal{B}], \tag{27}$$

$$L[\sigma,\mathcal{A},\mathcal{B}] \equiv -\bar{g}^{\mu\nu}(\nabla_\mu\sigma_a)^*\nabla_\nu\sigma_a - \frac{\chi_2}{4g^2}\bar{F}^2(\mathcal{A}) - \frac{\chi_2}{4g'^2}\bar{F}^2(\mathcal{B}), \tag{28}$$

i.e. the $SU(2)\times U(1)$ iso-doublet scalar σ becomes massless.

For large positive values of φ we obtain $A(\varphi,\sigma) \simeq A_{(+)} = 1$, $B(\varphi) \simeq B_{(+)} = 0$, and:

$$U_{\text{eff}}(\varphi,\sigma) \simeq U_{(+)}(\sigma) \equiv \frac{1}{4\chi_2 M_2}(m_0^2\sigma_a^*\sigma_a - M_1)^2 + 2\Lambda_0, \tag{29}$$

so that the effective matter Lagrangian (20) becomes (using notation (28)):

$$L^{(+)}_{\text{eff}} = X - U_{(+)}(\sigma) + L[\sigma,\mathcal{A},\mathcal{B}]. \tag{30}$$

In the case of absent $SU(2)\times U(1)$ scalar and gauge fields, a case already discussed in detail in Ref. 6, the purely "inflaton" effective potential

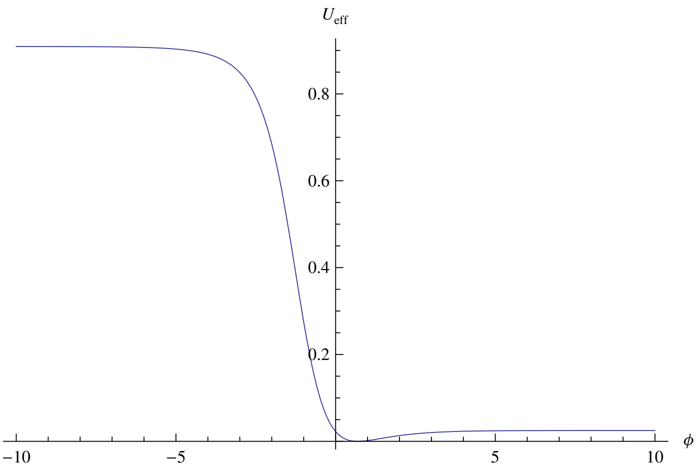

Fig. 1. Qualitative shape of the effective "inflaton" potential $U_{\rm eff}(\varphi)$ as function of φ, $M_1 > 0$.

$U_{\rm eff}(\varphi) = (f_1 e^{-\alpha\varphi} - M_1)^2[4\chi_2(f_2 e^{-2\alpha\varphi} + M_2)]^{-1}$ possesses another infinitely large flat region for large positive values of φ: $U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv M_1^2(4\chi_2 M_2)^{-1}$ and the shape of $U_{\rm eff}(\varphi)$ is depicted in Fig. 1.

It was found in Ref. 6 that this effective "inflaton" potential with two infinitely large flat regions with vastly different scales upon appropriate choice for the values of the parameters accommodates the description simultaneously of the "early" universe evolution (the inflationary epoch) corresponding to the first flat region for large negative φ-values (the left one in Fig. 1), as well as the "late" post-inflationary (dark energy dominated) universe corresponding to the second flat region for large positive φ-values (the right one in Fig. 1).

The presence of the $SU(2) \times U(1)$ scalar and gauge fields will not affect significantly the inflationary dynamics in the "early" universe (when φ runs on the left higher flat region of $U_{\rm eff}(\varphi)$, Fig. 1), and where the $SU(2) \times U(1)$ iso-doublet σ-field is massless, see (28) — this dynamics is governed by the purely kinetic "k-essence" "inflaton" part $A_{(-)}X + B_{(-)}X^2 - U_{(-)}$ of (27).

However, in the post-inflationary epoch (when φ runs on the right lower flat region of $U_{\rm eff}(\varphi)$,Fig. 1) the presence of the $SU(2) \times U(1)$ iso-doublet σ contributes very significantly to the full effective scalar potential (29), which emerges in an exactly Higgs-like form:

$$U_{(+)}(\sigma) = \frac{1}{4\chi_2 M_2}(m_0^2 \sigma_a^* \sigma_a - M_1)^2 + 2\Lambda_0 \equiv \lambda(\sigma_a^*\sigma_a)^2 - \mu^2 \sigma_a^*\sigma_a + \text{const.}, \quad (31)$$

$$\lambda = \frac{m_0^4}{4\chi_2 M_2}, \quad \mu^2 = \frac{M_1 m_0^2}{2\chi_2 M_2}. \quad (32)$$

Spontaneous $SU(2) \times U(1)$ symmetry breakdown occurs at the vacuum value:

$$|\sigma_{\rm vac}| = \frac{1}{m_0}\sqrt{M_1}, \quad (33)$$

with a small residual cosmological constant Λ_0, which is identified with the current epoch observable cosmological constant. Let us also note that in the post-inflationary epoch according to (30) and (31) the massless "inflaton" φ (entering the kinetic term X (21) only) loses all interactions with the $SU(2) \times U(1)$ iso-doublet σ (which has now become the Higgs field), thus avoiding undesirable fifth-force problems.

The dependence of the Higgs-like parameters (32) and the Higgs-like v.e.v. (33) on the integration constants $M_{1,2}$ and χ_2 explicitly reveals the nature of the Higgs-like spontaneous gauge symmetry breaking (31) as a gravity-assisted dynamically generated phenomenon in the post-inflationary epoch. It is triggered exclusively through the special non-Riemannian volume-form dynamics in the original gravity-gauge-field-matter action (1) leading to the remarkable two-flat-region shape of the "inflaton" effective potential (Fig. 1). Let us stress again that in the "early" universe there is no spontaneous breaking of $SU(2) \times U(1)$ gauge symmetry and the Higgs-like iso-doublet scalar field σ is massless there (27)–(28).

We conclude with a note on the plausible numerical values for some of the parameters involved. In Ref. 6 it was argued that it is natural to associate $M_1 \sim M^4_{\rm EW}$, $M_2 \sim M^4_{\rm Pl}$ and $\chi_2 \sim 10^{-1}$, where $M_{\rm EW}$ and $M_{\rm Pl}$ are the electroweak and Planck scales, respectively. Then, from (33) we see that it is natural also to associate the bare mass of the $SU(2) \times U(1)$ iso-doublet σ-field $m_0 \sim M_{\rm EW}$, so that we will have for the Higgs-like v.e.v. $|\sigma_{\rm vac}| \sim M_{\rm EW}$ conforming to the standard electroweak phenomenology.

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Appendix A. Canonical Hamiltonian Treatment of Gravity-Matter Theories with Non-Riemannian Volume-Forms

Let us briefly discuss the application of the canonical Hamiltonian formalism to the gravity-matter model based on two non-Riemannian spacetime volume-forms (1). For convenience we introduce the following short-hand notations for the field-strengths (2), (10) of the auxiliary 3-index antisymmetric gauge fields (the dot indicating time-derivative):

$$\Phi_1(A) = \dot{\mathcal{A}} + \partial_i \mathcal{A}^i, \quad \mathcal{A} = \frac{1}{3!}\varepsilon^{ijk} A_{ijk}, \quad \mathcal{A}^i = -\frac{1}{2}\varepsilon^{ijk} A_{0jk}, \tag{A.1}$$

$$\Phi_2(B) = \dot{B} + \partial_i B^i, \quad B = \frac{1}{3!}\varepsilon^{ijk} B_{ijk}, \quad B^i = -\frac{1}{2}\varepsilon^{ijk} B_{0jk}, \tag{A.2}$$

$$\Phi(H) = \dot{H} + \partial_i H^i, \quad H = \frac{1}{3!}\varepsilon^{ijk} H_{ijk}, \quad H^i = -\frac{1}{2}\varepsilon^{ijk} H_{0jk}. \tag{A.3}$$

The canonical momenta conjugated to (A.1)–(A.3) read:

$$\pi_A = R + L^{(1)} - \frac{4\Lambda_0}{\sqrt{-g}}(\dot{A} + \partial_i A^i), \tag{A.4}$$

$$\pi_B = L^{(2)} + \frac{1}{\sqrt{-g}}(\dot{H} + \partial_i H^i), \quad \pi_H = \frac{1}{\sqrt{-g}}(\dot{B} + \partial_i B^i), \tag{A.5}$$

and:

$$\pi_{A^i} = 0, \quad \pi_{B^i} = 0, \quad \pi_{H^i} = 0. \tag{A.6}$$

The latter imply that A^i, B^i, H^i are in fact Lagrange multipliers for certain first-class Hamiltonian constraints (see Eqs. (A.8)–(A.9)).

Using the short-hand notation $(u, \dot{u})$ to collectively denote the set of the basic gravity-matter canonical variables $(u) = \left(g_{\mu\nu}, \varphi, \sigma, \mathcal{A}^A_\mu, \mathcal{B}_m\right)$ and their respective velocities, we have for their respective canonically conjugated momenta:

$$p_u = (\dot{A} + \partial_i A^i)\frac{\partial}{\partial \dot{u}}(R + L^{(1)}) + (\dot{B} + \partial_i B^i)\frac{\partial}{\partial \dot{u}} L^{(2)}. \tag{A.7}$$

Now, using relations (A.4)–(A.7) we arrive at the following Dirac-constrained canonical Hamiltonian of the model (1):

$$\mathcal{H}_{\rm can} = p_u\, \dot{u} - \frac{\sqrt{-g}}{8\Lambda_0}(R + L^{(1)} - \pi_A)^2 - \pi_H\sqrt{-g}L^{(2)} \\ + \sqrt{-g}\pi_H\pi_B - \partial_i A^i \pi_A - \partial_i B^i \pi_B - \partial_i H^i \pi_H. \tag{A.8}$$

The last three terms in (A.8) show that, indeed, the auxiliary gauge field components A^i, B^i, H^i (with vanishing canonically conjugated momenta (A.6)) are Lagrange multipliers for the Dirac first class constraints:

$$\pi_A = -M_1 = \text{const.}, \quad \pi_B = -M_2 = \text{const.}, \quad \pi_H = \chi_2 = \text{const.}, \tag{A.9}$$

which are the canonical Hamiltonian counterparts of Lagrangian constraint equations of motion (15).

To conclude, the canonical Hamiltonian treatment of (1) reveals the (almost) pure-gauge nature of the auxiliary 3-index antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ — building blocks of the non-Riemannian spacetime volume-form formulation of the modified gravity-matter model (1). Namely, the canonical momenta π_A, π_B, π_H conjugated to the "magnetic" parts A, B, H (A.1)–(A.3) of the auxiliary 3-index antisymmetric tensor gauge fields are constrained through Dirac first-class constraints (A.9) to be constants identified with the arbitrary integration constants χ_2, M_1, M_2 (15) arising within the Lagrangian formulation of the model. The canonical momenta $\pi^i_A, \pi^i_B, \pi^i_H$ conjugated to the "electric" parts

A^i, B^i, H^i (A.1)–(A.3) of the auxiliary 3-index antisymmetric tensor gauge field are vanishing (A.6), which makes the latter canonical Lagrange multipliers for the above Dirac first-class constraints.

Thus, the method of employing non-Riemannian spacetime volume-forms in gravity-matter theories does not introduce any additional propagating field-theoretic degrees of freedom apart from the standard ones.

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